January 7, February 3,13, 2020

DRAFT

Newton on Universal Gravitation

John D. Norton
http://www.pitt.edu/~jdnorton

Chapter for a book provisionally titled
The Large Scale Structure of Inductive Inference

1. Introduction

The argumentation in Newton's seventeenth century *Mathematical Principles of Natural Philosophy* remains to this day a model of tight, carefully controlled argumentation. Its inductive centerpiece lays out the evidential case for his theory of universal gravitation with exemplary caution and discipline. Within his argumentation, there are two cases of pairs of propositions in which relations of inductive support cross over each other, in analogy to the relations of structure support in an arch. The first pair comprises the two core propositions of Newton's celebrated "moon test". The second pair comprises the propositions of an inverse square law of gravity and of the elliptical orbits of the planets.

In both cases, the individual relations of support have the following structure: the observed evidence supports a proposition by means of a warranting hypothesis. Schematically, this can be we written

Observed evidence
(warrant) Hypothesis
________________(deduce)
Proposition

The crossing over of relations of support arises in both cases in the following way. We have two propositions, proposition_1 and proposition_2, such that

Observed evidence

(warrant) Proposition$_1$

_______________(deduce)

Proposition$_2$

Observed evidence

(warrant) Proposition$_2$

________________(deduce)

Proposition$_2$

Finally, each of the individual inferences above is deductive. They combine to give a totality in which the observed evidence inductively supports both propositions. That is, the relations of support are locally deductive but inductive in their combination.

Observed evidence

_____________(induction)

Proposition$_1$ & Proposition$_2$

The two examples are treated in turn in the sections that follow.

2. The Moon Test

One of Newton's more remarkable discoveries in his theory universal gravitation is the identity of two forces. The first is the celestial force that deflects planets into orbit around the sun and deflects moons into orbits around their planets. The second is the force of gravity that leads to the fall of free bodies at the earth's surface, such as hurled stones. That these forces are the same is now a commonplace. It was a major discovery in the seventeenth century, for the ancient tradition had been that the physics of terrestrial bodies differs from the physics of celestial matter. Newton needed a strong argument to establish the identity.

The identity of the two forces was established early by Newton in Book III of his *Principia* (1726). That book presents a sequence of propositions of that lays out his argument for universal gravitation. The first three propositions establish that the celestial force of attraction acting on an orbiting body varies with the inverse square of distance from the center of the attracting body in three cases: the orbit of Jupiter's moons about the center of Jupiter, the orbit of the planets about the sun's center and the orbit of the moon about the earth's center. The fourth proposition asserts the identity of terrestrial gravity and the celestial force acting on the earth's moon.

To arrive at this fourth proposition, Newton determined the acceleration of the moon towards the earth. It is this acceleration that deflects the moon from its linear, inertial motion and brings it into orbit around the earth. We would now represent this acceleration directly as so many feet/second^2 or meters/second^2. Newton proceeded indirectly. A body falling with constant acceleration a from rest will cover a distance $at^2/2$ in time t. Newton used this distance as the measure of acceleration.

As a result of its orbital motion, Newton noted that the moon falls 15 Paris feet 1 inch 1 4/9 lines [twelfths inch] in one minute. That is, it falls 15.0934 Paris feet in one minute. The moon is roughly 60 times farther away from the center of the earth than a point on the earth's surface. Hence, if the celestial force acting on the moon is governed by an inverse square law all the way down to the earth's surface, it would be 60^2 times greater on the earth's surface. That means that a body falling under its action at the earth's surface would fall 15.0934 x 60^2 Paris feet in one minute. One minute is a time unfamiliar in our experience for bodies to fall above the surface of the earth. So Newton scaled the time of fall to one second. Conveniently, one second is 1/60th minute. Since the distance fallen varies with the square of time t, a body falling under the celestial force at the earth's surface for one second would fall $1/60^2$ of 15.0934 x 60^2 Paris feet, that is, 15.0934 Paris feet. This matches well how bodies fall on the surface of the earth under gravity, as measured by experiments on pendula. Newton (1726, p. 408) concluded:

> And therefore the force by which the Moon is retained in its orbit becomes, at the very surface of the Earth, equal to the force of gravity which we observe in heavy bodies there. And therefore (by Rule 1 & 2) the force by which the Moon is retained in its orbit is that very same force which we commonly call gravity; for were gravity another force different from that, then bodies descending to the Earth with the joint impulse of both forces would fall with a double velocity…

The case Newton made here is a powerful one. In recollections recorded much later, Newton asserted that he found the arguments of these first four propositions in 1666. He noted (1888, p. xviii) of the moon test:

> At the same year [1666] I began to think of gravity extending to the orbit of the Moon, … and thereby compared the force requisite to keep the Moon in her orb with the force of gravity at the surface of the earth and found them answer pretty nearly.

3. The Inferences Summarized

The inference above can be summarized as follows:

Observed acceleration of fall of terrestrial bodies and the moon.

(warrant) $H_{inv. square}$: The celestial force acting on the moon is strengthened by an inverse square law with distance at the earth's surface.

____(deduce)

Intermediate conclusion: Equality of accelerations at the earth's surface due to gravity and the celestial force.

(warrant) Rules 1 and 2 of Newton's Rules of Reasoning in Philosophy

____

$H_{identity}$: Terrestrial gravitation and the lunar celestial force are the same.

The last step might seem superfluous. Newton has found that the acceleration due to gravity and the celestial force match at the earth's surface. Is that not enough to show the identity of the two forces? It is very close, but there is a loophole. It might just be that the force of gravity does not act on celestial matter such as comprises the moon; and that the celestial force does not act on ordinary, terrestrial matter. Newton closed the gap with the rules of reasoning he had declared earlier in *Principia*. The relevant idea is that we are to assign the same cause to the same effect. I will not pursue this use of the rules further. In *The Material Theory of Induction* (ms), Ch. 6 Simplicity, I described my discomfort with the rules and indicated how they can be replaced in this case by a simple material fact: that the matter of the moon would behave like terrestrial matter were it brought to the earth's surface. What results is the simpler inference:

Observed acceleration of fall of terrestrial bodies and the moon.

(warrant) $H_{inv. square}$: The celestial force acting on the moon is strengthened by an inverse square law with distance at the earth's surface.

____(deduce)

Intermediate conclusion: Equality of accelerations at the earth's surface due to gravity and the celestial force.

(warrant) Terrestrial and lunar matter respond to the same forces.

____(deduce)

$H_{identity}$: Terrestrial gravitation and the lunar celestial force are the same.

For present purposes, what matters is that the inverse square law, $H_{inv. square}$, is used as part of the inference to the identity result, $H_{identity}$. This usage forms one half of the arch shown below in Figure 3.

There is a second inference here that Newton does not make explicit. He has inferred that the celestial force is governed by an inverse square law in other parts of the solar system. But how does he know that this inverse square dependence on distance will continue to hold when he moves out of the celestial realm down to the terrestrial realm? It is striking that the inference sketched above works so well. That the two forces "answer pretty nearly" as Newton remarked gives one confidence that the inverse square law, introduced as an hypothesis above, is also supported by the successful outcome. Perhaps this is why Newton reported the agreement as a memorable phase in his discovery of universal gravitation. Though not given explicitly by Newton, we can summarize this naturally suggested argument as follows:

Observed acceleration of fall of terrestrial bodies and the moon.

(warrant) $H_{identity}$: Terrestrial gravitation and the lunar celestial force are the same.

____(deduce)

Intermediate conclusion: Celestial/gravitational accelerations at the earth's surface and the moon's orbit are in the ratio of an inverse square of distances to the earth's center.

(warrant) Terrestrial and lunar matter respond to the same forces.

____(deduce)

$H_{inv.\ square}$: The celestial force acting on the moon is strengthened by an inverse square law with distance at the earth's surface.

This second inference forms the second half of the relations of support displayed in Figure 3.

For our purposes, we have two inferences each of whose conclusions is used as a warrant in the argument for the other. We can draw the corresponding arch as Figure 3.

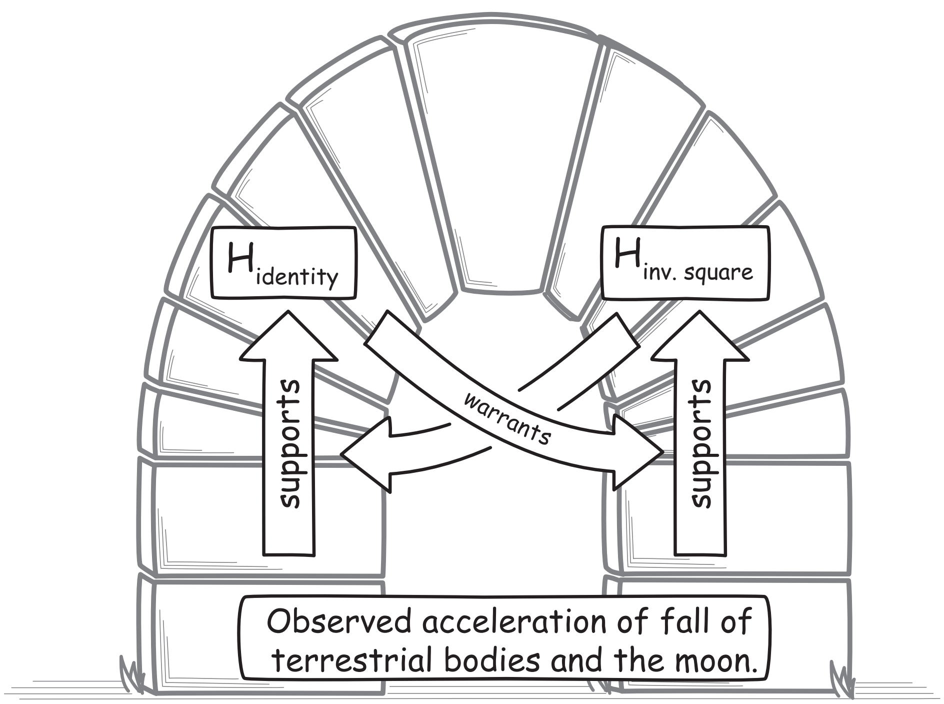

Figure 3. The Arch for the Moon Test.

While the component relations of support are deductive, the combined result is that the observed accelerations provide inductive support for the two hypotheses:

Observed acceleration of fall of terrestrial bodies and the moon.

____(induction)

$H_{identity}$: Terrestrial gravitation and the lunar celestial force are the same.

$H_{inv.\ square}$: The celestial force acting on the moon is strengthened an
inverse square law with distance at the earth's surface.

4. Elliptical Orbits and the Inverse Square Law

The next pair of mutually supporting propositions assert the planets move along elliptical orbits and that their motion is governed by an inverse square law of gravity. Planetary astronomy poses a curve-fitting problem. We have many observed positions of the planets. Which curve do we fit to them to recover their orbits? Prior to Newton, Kepler had found that elliptical orbits could be fitted to the observed positions of the planets. This result came to be known later as "Kepler's Second Law." It is, for example, so-called in Maxwell's, *Matter and Motion* (1894), p. 110. From it one can infer that each planet is attracted to the sun by a force that varies inversely with the square of distance from the sun, as the planet moves through its orbit. That an elliptical motion is associated with this inverse square law is an early result proved by Newton in Book I of *Principia* (Proposition XI. Problem VI.) Maxwell (1894, p. 112) uses this result to infer from the elliptical motions of the planets to the inverse square law of gravity. He concludes:

> Hence the acceleration of the planet is in the direction of the sun, and is inversely as the square of the distance from the sun. This, therefore, is the law according to which the attraction of the sun on a planet varies as the planet moves in its orbit and alters its distance from the sun.

That is we have the following inference:

Observed positions of the planets.

(warrant) $H_{ellipses}$: The planets move in their specific elliptical orbits.

____(deduce)

$H_{inv.\ square}$: The planets are attracted to the sun by a force that varies with the inverse square of distance.

Newton himself, however, was more circumspect. This relation of support is straightforward only in so far as we assume that the fit of an ellipse to the observed motions is exact. Newton knew that it was not exact, so he did not offer Maxwell's inference in his *Principia*. That an elliptical motion is governed by an inverse square force law is merely reported as a theorem of mathematics.

In its place, Newton offered an inverted relation of support. The pertinent discussion comes later in Book III in his Proposition XIII. Theorem XIII. At this stage in the development, Newton has already inferred the inverse square law of gravity from other phenomena. He will now infer from the inverse square law to the elliptical motions of the planets. Noting the inversion explicitly, he wrote:

> Now that we know the principles on which they [the motions of the planets] depend, from these principles we deduce the motions of the heavens *a priori*. Because the weights of the planets towards the sun are inversely as the squares of their distances from the sun's centre, if the sun were at rest, and the other planets did not act one upon another, their orbits would be ellipses, having the sun in their common focus;…

Newton here offers a relation of support that inverts the one given above by Maxwell:

Observed positions of the planets.

(warrant) $H_{inv.\ square}$: The planets are attracted to the sun by a force that varies with the inverse square of distance.

____(deduce)

$H_{ellipses}$: The planets move in their specific elliptical orbits.

The observed positions of the planets are still needed as a premise in the inference since an inverse square law of attraction from the sun is also compatible with parabolic and hyperbolic trajectories. These are ruled out by the period motion of the planets. Then specific positions of the planets at specific times are needed to recover the specific ellipse that is the orbit of each planet.

Newton's inference, however, is qualified by an idealization indicated in his remark above: "… if the sun were at rest, and the other planets did not act one upon another..." The orbits of the planets are not exactly elliptical because of perturbations from the gravitational attraction of the other planets. These deviations are generally negligible at the level of accuracy of Newton's analysis. However, a noticeable perturbation was produced by the massive planet acting on the motion of Saturn.[1] It is greatest when the two planets are nearest each other, that is, when they are in conjunction. "And hence arises," Newton concluded, "a perturbation of the orbit of Saturn in every conjunction of this planet so sensible, that astronomers are puzzled with it."

5. The Exactness of the Inverse Square Law

Newton did not explicitly incorporate the inference from the elliptical orbits of the planet to the inverse square law in the carefully developed sequence of propositions in Book III of *Principia*. However an important step in that sequence was something quite close to this inference. It concerned the inverse square law of gravity. How does Newton know that this is the correct law, exactly? Might another, similar law work as well or even better? Does gravity conform with the inverse square law only as an approximation? Perhaps the force varies with distance r according to $1/r^{2+\delta}$, where δ is some small number close to zero?

In one of the most brilliant analyses of his *Principia*, Newton showed that we have strong evidence for the force of attraction conforming exactly with the inverse square law. Under such a law, Newton had shown, the unperturbed planets move along an elliptical path that is fixed in space. The aphelion of each planet—the point of greatest distance from the sun—will be fixed in space and the planet will return to it after a complete circuit of 360° around the sun. The ellipse's

[1] Less noticeable, Newton reported, were the perturbations in Jupiter's motion due to the attraction of Saturn. He reported other perturbations as "yet far less." The exception was the sensible disturbance to the orbit of the earth due to the moon.

major axis, the line of the apsides connecting aphelion and perihelion, would be correspondingly fixed.

This fixity would be lost, Newton now showed, if the law differed from an inverse square law. In Proposition 45, Corollary 1 of Book I, Newton considered the case of bodies orbiting in near circular orbits. He showed that if the law of attraction differed from an inverse square law, then a planet would not return to its aphelion after a circuit of 360^o around the sun. It would need to complete more or less of the circuit according to how much the force deviated from an inverse square law. That is, for a $1/r^{2+\delta}$ force law, the planet would return to its aphelion after passing $360^o/\sqrt{1-\delta}$. The result was remarkably robust, holding even when the deviation from the inverse square law δ was not small.

Since our planets do move in near circular orbits, Newton could apply his result to the motions of the planets. Setting aside known perturbations, the planets do trace out fixed elliptical orbits, returning to their aphelia after a 360^o circuit around the sun. Newton could conclude with satisfaction in Book III, Proposition II Theorem II:

> [The inverse square law] is, with great accuracy, demonstrable from the quiescence of the aphelion points; for a very small aberration from the proportion according to the inverse square law of the distances would (by Cor. 1, Prop. XLV, Book I) produce a motion of the apsides sensible enough in every single revolution, and in many of them enormously great.

In summary form, this argument is a version of Maxwell's argument, since it infers from a property of the elliptical orbits of the planets to the exact inverse square law of gravity:

Observed positions of the planets.
(warrant) $H_{ellipses}$: The planets move in their specific elliptical orbits.
Newton's Proposition 45, Corollary 1, Book I.
____(deduce)
$H_{inv.\ square}$: The Planets are attracted to the sun by a force that varies with the inverse square of distance.

The overall structure of the relations of support displayed here is of the two hypotheses accruing support from the observed positions of the planets over time. While the two component inferences are deductive, the combined relations of support are inductive and can be summarized as

Observed positions of the planets.

____(induction)

$H_{ellipses}$: The planets move in their specific elliptical orbits.

$H_{inv.\ square}$: The Planets are attracted to the sun by a force that varies with the inverse square of distance.

In broad strokes, the relations of support recounted here in Sections 4 and 5 are among the two hypotheses $H_{inv.\ square}$ and $H_{ellipses}$. They enter into the mutual relations of support pictured in the arch analogy of figure 4.

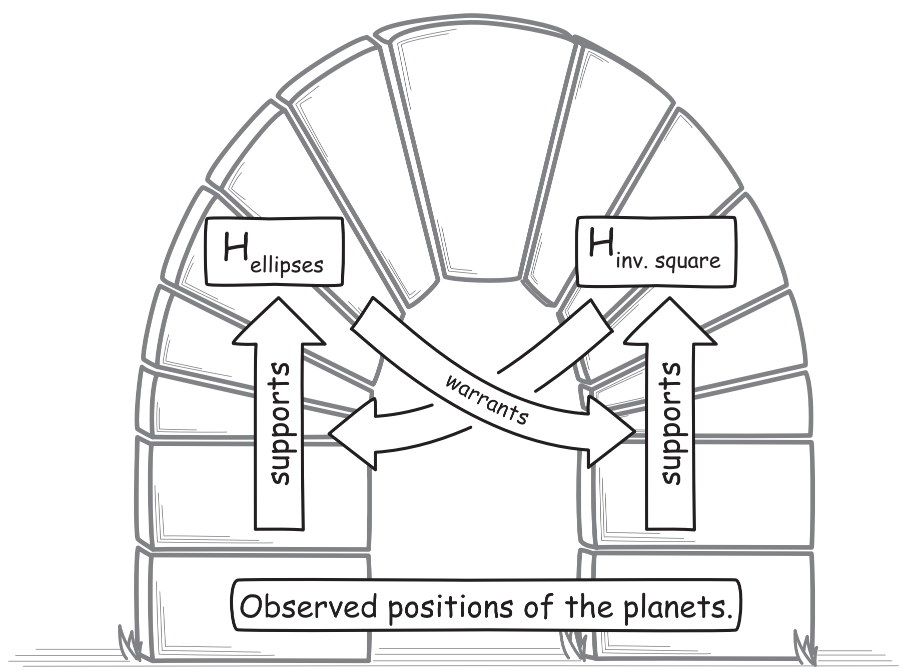

Figure 4. Elliptical Orbits and the Inverse Square Law

6. Conclusion

We have seen here, two pairs of propositions in Newton's *Principia* that mutually support one another. A close reading of Newton's text is quite likely to reveal more. A natural candidate is Kepler's harmonic rule that relates the period and mean radii of planetary and lunar orbits: $(\text{period})^2$ is directly proportional to $(\text{radius})^3$. Newton infers from this harmonic rule to his inverse square law. We now routinely invert the inference and infer from the inverse square law to the harmonic law.

Such inversions are encouraged by a development common in maturing theories. We are initially inclined to infer from the elliptical orbits of the planets to the inverse square law of attraction, for the elliptical orbits are closer to observations. As the theory matures, we find multiple supports for the inverse square law. We are also recognize that Newton's fully elaborated system corrects the simple statement that the planets move in ellipses; for in some cases, the perturbing effects of other celestial bodies move them away from their ellipses. Then it becomes more natural to invert the relation of support and see the inverse square law as supporting a corrected version of the original observations of elliptical orbits.

Another example of this inversion is found in the role of atomic spectra in foundation of quantum theory, as related in the chapter on atomic spectra. Ritz's combination principle supports the discrete energy levels of Bohr's 1913 theory of the atom and thus the quantum theory that developed from it. The developed quantum theory, however, entails a version of the Ritz principle, corrected by selection rules. This complication indicates the inverted relation of support.

References

Maxwell, James. Clerk (1894) *Matter and Motion*. London: Society for Promoting Christian Knowledge.

Newton, Isaac (1726), *Mathematical Principles of Natural Philosophy*. 3rd ed. Trans. Andrew Motte, rev. Florian Cajori. University of California Press, 1962.

Newton, Isaac (1888) *A Catalogue of the Portsmouth Collection of Books and Papers Written by or Belonging to Sir Isaac Newton*. Cambridge: Cambridge University Press.

Norton, John D (ms) *The Material Theory of Induction*.
http://www.pitt.edu/~jdnorton/homepage/cv.html#manuscripts