Homework Problems: The following problems are to be completed and turned in by the beginning of class on the due date.

1. Surf to URL http://www.falstad.com/circuit/ and run the Java circuit simulation applet. Note that there are many examples to play with under the Circuits menu. Choose Circuits → Basics → Resistors for this Homework problem.

This circuit contains two sets of triple parallel branches. Let the switches in the circuit be labeled as $s_1, s_2, ..., s_6$, starting at the top left and proceeding left to right across the top row, and then across the bottom row. Therefore $s_6$ denotes the switch at the bottom right.

Let $s_1 ... s_6$ take binary values to denote switch positions, where 0 denotes open and 1 denotes closed. We can represent any possible switch state of the circuit by this 6-bit binary word.

(a) Normally, when this circuit is initialized, the switch state is 101010. If it is not, change the switches accordingly. Perform elementary circuit analysis to derive all voltages and currents for the circuit in this switch state, and thereby verify the numerical values given by the applet. Show all calculations.
(b) Repeat part (a) for states 011011 and 011001.
(c) Let $I_5$ denote the current passing through the 200$\Omega$ resistor, and define a binary output $X$, $X = \begin{cases} 1 & I_5 \leq 2 \text{mA} \\ 0 & I_5 > 2 \text{mA} \end{cases}$

Therefore, this circuit has six inputs $s_1 ... s_6$ and two outputs: $I_5$ and $X$. Use Excel, Matlab or some other computer tool to prepare and print the truth table for the circuit, showing both the current $I_5$ and logical output $X$ for each possible input state. (You do not have to show the calculations for each input state.)

2. The positional number systems we have discussed in class allow one to represent whole numbers in different bases, but do not allow for the representation of fractions. A simple extension of these ideas includes whole numbers and fractions using a fixed-point representation. For example, everyone understands that when using a decimal representation such as 26.125, each digit to the left of the decimal point represents a positive power of ten, and each digit to the right of the decimal point represents a negative power of ten:

$$26.125_{10} = \left(2\times10^1\right) + \left(6\times10^0\right) + \left(1\times10^{-1}\right) + \left(2\times10^{-2}\right) + \left(5\times10^{-3}\right)$$
The same idea can be applied to representation in any base. For a representation in base \( b \), let the digits be \( d_{i}d_{i}d_{i}d_{i}d_{i}d_{i}d_{i}d_{i}d_{i}d_{i}d_{i}d_{i}d_{i}d_{i}d_{i}d_{i}d_{i}d_{i} \). Then we have:

\[
d_{i}d_{i}d_{i}d_{i}d_{i}d_{i}d_{i}d_{i}d_{i}d_{i}d_{i}d_{i}d_{i}d_{i}d_{i}d_{i}d_{i}d_{i} = (d_{2} \times b^{2}) + (d_{1} \times b^{1}) + (d_{0} \times b^{0}) + (d_{-1} \times b^{-1}) + (d_{-2} \times b^{-2}) + (d_{-3} \times b^{-3})
\]

So for example, the binary fixed-point number 11010.001 can be converted to decimal as follows:

\[
11010001_{2} = (1 \times 2^{4}) + (1 \times 2^{3}) + (0 \times 2^{2}) + (1 \times 2^{1}) + (0 \times 2^{0}) + (0 \times 2^{-1}) + (1 \times 2^{-3}) = 16 + 8 + 2 + \frac{1}{8} = 26.125
\]

Convert each of the following numbers to the indicated base. Show all steps of the conversion.

(a) E0CD.35  
   hexadecimal to decimal 
(b) 620.236  
   octal to hexadecimal 
(c) 101011.011  
   binary to decimal 
(d) 769.2  
   decimal to hexadecimal

3. Perform the following arithmetic in the indicated base. Show all work. Do not convert to a different base before performing the operation.
   (a) Hexadecimal Addition:  FACE + CAFE
   (b) Binary Division:  11001110010 \div 10110
   (c) Octal Addition:  4.64 + 3.36
   (d) Octal Multipication:  4.64 \times 3.36

4. Repeat Problem 3, by converting each operand to decimal, carrying out the operation in decimal arithmetic, and then converting the answer back to the original base. Show all steps of the conversions.

5. Let \( X \) be an octal number with seven digits, denoted \( abcedefg \). Furthermore, all of the digits are different, none of the digits are zero, and they have the following properties:
   (i) \( X \) is a multiple of 7.
   (ii) The six-digit octal number \( abcd ef \) is a multiple of 6.
   (iii) The five-digit octal number \( abcd e \) is a multiple of 5.
   (iv) The four-digit octal number \( abcd \) is a multiple of 4.
   (v) The three-digit octal number \( abc \) is a multiple of 3.
   (vi) The two-digit octal number \( ab \) is a multiple of 2.

Determine all solutions for \( X \). (Hint: Start with the last property.)

Of course, it is possible to write a computer program to exhaustively search for all solutions, but such solutions will receive no credit. You must solve this problem using only what you know about number representations and divisibility, and careful reasoning.
Recitation Problems: The following problems are not to be turned in. Instead, the solutions to these problems will be presented at the recitations. You should try these problems on your own before coming to recitation.

1. Solve Homework Problem 1, part (a) for switch state 010110.


3. Perform the octal multiplication $262_8 \times 155_8$ directly in base 8. Then perform the computation again by converting each operand to decimal, carrying out the multiplication in decimal, and converting the product back to octal. Confirm that your answers are identical.