International Business Cycle Models

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Overview

In this lecture, we will cover various international business cycle models

- Endowment model with complete markets and financial autarky

  Cole and Obstfeld (1991)

- One good production model with complete markets and non-contingent bonds

  Baxter and Crucini (1995)

- Two good production model with complete markets

  Backus, Kehoe, and Kydland (1994)

- Two good production model with financial autarky

  Heathcote and Perri (2002)

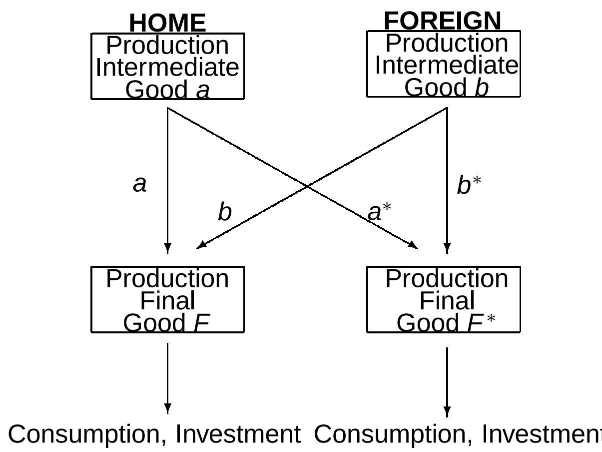
- One good production model with endogenous incomplete markets

  Kehoe and Perri (2002)

1 Heathcote and Perri (2002)

Consider a two country, two good, real business cycle model (Backus, Kehoe, and Kydland 1994). Countries specialize in production of one intermediate good each. Final goods are bundles of both intermediate goods, and are used for consumption and investment. There is home bias in production of final goods, and the substitutability between domestic and foreign intermediate goods is given by σ. The law of one price holds.

I-firms in the home country produce good a, while those in the foreign country produce good b. Both goods are used in production of final good (f-firms), which is used for consumption and investment.



1.1 Households

Preferences of representative household

$$\sum_{t=0}^{\infty}\sum_{s^t}\pi(s^t)\beta^t U\left(c_t(s^t), 1-n_t(s^t)\right)$$

where $U(c, 1-n) = \dfrac{1}{1-\gamma}\left[c^{\mu}(1-n)^{1-\mu}\right]^{1-\gamma}$.

Households supply labor and rent capital to competitive intermediate goods-producing firm (i-firms). The law of motion of capital is given by

$$k_{t+1} = (1-\delta)k_t + x_t.$$

1.2 Intermediate goods

I-firms's static maximization problem is given by

$$\begin{array}{cc} \max\limits_{k_t, n_t} & F(z_t, k_t, n_t) - w_t n_t - r_t k_t \\ \text{s.t.} & k_t, n_t \geq 0 \end{array} .$$

Each country specializes in production of a single good (i-firms):

$$\begin{aligned} a_t + a_t^* &= F(z_t, k_t, n_t) \equiv z_t k_t^{\alpha} n_t^{1-\alpha} \\ b_t + b_t^* &= F(z_t^*, k_t^*, n_t^*) \equiv z_t^* k_t^{*\alpha} n_t^{*1-\alpha} \end{aligned}$$

1.3 Final goods

F-firms's static maximization problem is given by

$$\begin{array}{ll} \max_{k_t, n_t} & G(a_t, b_t) - q_t^a a_t - q_t^b b_t \\ \text{s.t.} & a_t, b_t \geq 0 \end{array} .$$

Resource constraints (f-firms):

$$\begin{aligned} c_t + x_t &= G(a_t, b_t) \equiv \left[\omega_1 a_t^{\frac{\sigma-1}{\sigma}} + \omega_2 b_t^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \\ c_t^* + x_t^* &= G^*(a_t^*, b_t^*) \equiv \left[\omega_2 a_t^{*\frac{\sigma-1}{\sigma}} + \omega_1 b_t^{*\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \end{aligned}$$

with $\omega_1 > \omega_2$ representing home bias

1.4 Budget contraints

Complete markets (BKK 1994)

$$\begin{gathered} c(s^t) + x(s^t) + q^a(s^t) \textstyle\sum_{s_{t+1}} Q(s^t, s_{t+1}) B(s^t, s_{t+1}) \\ = q^a(s^t) \left(r(s^t) k(s^t) + w(s^t) n(s^t) \right) + q^a(s^t) B(s^{t-1}, s_t) \end{gathered}$$

Bond economy

$$\begin{gathered} c(s^t) + x(s^t) + q^a(s^t) Q(s^t) B(s^t) \\ = q^a(s^t) \left(r(s^t) k(s^t) + w(s^t) n(s^t) \right) + q^a(s^t) B(s^{t-1}) \end{gathered}$$

Financial autarky (Heathcote and Perri 2002)

$$c(s^t) + x(s^t) = q^a(s^t) \left(r(s^t) k(s^t) + w(s^t) n(s^t) \right)$$

1.5 Variables of interest

- Real exchange rate

$$e \equiv \frac{P_c}{P_c^*} \left(= \frac{q_a}{q_a^*} = \frac{q_b}{q_b^*} \because \text{law of one price} \right)$$

- Terms of trade

$$p \equiv \frac{q_b}{q_a} \left(= \frac{G_b}{G_a} \because \text{f-firm optimality} \right)$$

- Net exports (nominal)

$$nx = q^a a^* - q^b b$$

1.6 Equilibrium Conditions

From household optimization

$$
\begin{aligned}
-\frac{U_c\left(s^t\right)}{U_n\left(s^t\right)} &= q^a\left(s^t\right) w\left(s^t\right) \\
U_c\left(s^t\right) &= \sum_{s^{t+1}|s^t} \pi\left(s^{t+1}|s^t\right) \beta U_c\left(s^{t+1}\right)\left[q^a\left(s^{t+1}\right) r\left(s^{t+1}\right) - (1-\delta)\right]
\end{aligned}
$$

Complete markets

$$
\begin{aligned}
Q\left(s^t, s_{t+1}\right) &= \pi\left(s^{t+1}|s^t\right) \beta \frac{U_c\left(s^t, s_{t+1}\right)}{U_c\left(s^t\right)} \frac{q^a\left(s^t, s_{t+1}\right)}{q^a\left(s^t\right)} \\
Q\left(s^t, s_{t+1}\right) &= \pi\left(s^{t+1}|s^t\right) \beta \frac{U_c^*\left(s^t, s_{t+1}\right)}{U_c^*\left(s^t\right)} \frac{q_a^*\left(s^t, s_{t+1}\right)}{q_a^*\left(s^t\right)} \\
\Rightarrow \frac{U_c^*\left(s^t, s_{t+1}\right)}{U_c^*\left(s^t\right)} &= \frac{U_c\left(s^t, s_{t+1}\right)}{U_c\left(s^t\right)} \frac{e\left(s^t, s_{t+1}\right)}{e\left(s^t\right)}
\end{aligned}
$$

Bond economy

$$
Q\left(s^t\right) = \sum \pi\left(s^{t+1}|s_t\right) \beta \frac{U_c\left(s^t, s_{t+1}\right)}{U_c\left(s^t\right)} \frac{q^a\left(s^t, s_{t+1}\right)}{q^a\left(s^t\right)}
$$

Financial autarky

$$
c(s^t) + x(s^t) = y(s^t)
$$

From i-firms optimization

$$
\begin{aligned}
w\left(s^t\right) &= F_n\left(s^t\right) \\
r\left(s^t\right) &= F_k\left(s^t\right)
\end{aligned}
$$

From f-firms optimization

$$
\begin{aligned}
q^a\left(s^t\right) &= G_a\left(s^t\right) \\
q^b\left(s^t\right) &= G_b\left(s^t\right)
\end{aligned}
$$

Notes for linearization

- $\hat{x} = \frac{x - \bar{x}}{\bar{x}}$
- $\log(1 + \hat{x}) \approx \hat{x}$

- $f(x,y) \approx f(\bar{x},\bar{y}) + (x-\bar{x})f_x(\bar{x},\bar{y}) + (y-\bar{y})f_y(\bar{x},\bar{y})$

Let s denote the steady state share of locally produced intermediate goods in final goods production. Note that

$$s = \frac{\bar{a}}{\bar{a}+\bar{b}} = \frac{\bar{a}}{\bar{a}+\bar{a}^*} = \frac{\bar{q}\bar{a}}{\bar{G}}$$

From market clearing conditions for intermediate goods,

$$\begin{aligned} a + a^* &= F(z,k,n) \\ b + b^* &= F(z^*,k^*,n^*) \end{aligned}$$

$$\Downarrow$$

$$\begin{aligned} \frac{a-\bar{a}+a^*-\bar{a}^*}{\bar{a}+\bar{a}^*} &= \frac{F(z,k,n)-F(\bar{z},\bar{k},\bar{n})}{F(\bar{z},\bar{k},\bar{n})} = \hat{y} \\ \frac{b-\bar{b}+b^*-\bar{b}^*}{\bar{b}+\bar{b}^*} &= \frac{F(z^*,k^*,n^*)-F(\bar{z}^*,\bar{k}^*,\bar{n}^*)}{F(\bar{z}^*,\bar{k}^*,\bar{n}^*)} = \hat{y}^* \end{aligned}$$

$$\Downarrow$$

$$\frac{a-\bar{a}}{\bar{a}}\frac{\bar{a}}{\bar{a}+\bar{a}^*} + \frac{a^*-\bar{a}^*}{\bar{a}^*}\frac{\bar{a}^*}{\bar{a}+\bar{a}^*} = s\hat{a} + (1-s)\hat{a}^* = \hat{y} \quad (1)$$

$$\frac{b-\bar{b}}{\bar{b}}\frac{\bar{b}}{\bar{b}+\bar{b}^*} + \frac{b^*-\bar{b}^*}{\bar{b}^*}\frac{\bar{b}^*}{\bar{b}+\bar{b}^*} = (1-s)\hat{b} + s\hat{b}^* = \hat{y}^* \quad (2)$$

Using the f-firm FOCs

$$\begin{aligned} G_a &= q_a \\ G_b &= q_b \end{aligned}$$

$$\Downarrow$$

$$\begin{aligned} \frac{\sigma-1}{\sigma}\omega_1 a_t^{\frac{-1}{\sigma}} G^{\frac{1}{\sigma}} &= q_a \\ \frac{\sigma-1}{\sigma}\omega_2 b_t^{\frac{-1}{\sigma}} G^{\frac{1}{\sigma}} &= q_b \end{aligned}$$

$$\Downarrow$$

$$
\begin{aligned}
\log\left(\left(\frac{a_t}{\bar{a}}\right)^{\frac{-1}{\sigma}}\left(\frac{G}{\bar{G}}\right)^{\frac{1}{\sigma}}\right) &= \log\left(\frac{q_a}{\bar{q}_a}\right) \\
\log\left(\left(\frac{b_t}{\bar{b}}\right)^{\frac{-1}{\sigma}}\left(\frac{G}{\bar{G}}\right)^{\frac{1}{\sigma}}\right) &= \log\left(\frac{q_b}{\bar{q}_b}\right)
\end{aligned}
$$

$$\Downarrow$$

$$
\begin{aligned}
\hat{G} - \hat{a} &= \sigma \hat{q}_a && (3) \\
\hat{G} - \hat{b} &= \sigma \hat{q}_b && (4)
\end{aligned}
$$

Using production of f-firms

$$
\begin{aligned}
G(a_t, b_t) \equiv &\quad \left[\omega_1 a_t^{\frac{\sigma-1}{\sigma}} + \omega_2 b_t^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \\
G^*(a_t^*, b_t^*) \equiv &\quad \left[\omega_2 a_t^{*\frac{\sigma-1}{\sigma}} + \omega_1 b_t^{*\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}
\end{aligned}
$$

$$\Downarrow$$

$$
\begin{aligned}
G(a_t, b_t) &\approx G(\bar{a}, \bar{b}) + (a - \bar{a})G_a(\bar{a}, \bar{b}) + (b - \bar{b})G_b(\bar{a}, \bar{b}) \\
\frac{G(a_t, b_t) - G(\bar{a}, \bar{b})}{G(\bar{a}, \bar{b})} &= \frac{(a - \bar{a})G_a(\bar{a}, \bar{b}) + (b - \bar{b})G_b(\bar{a}, \bar{b})}{G(\bar{a}, \bar{b})} \\
\hat{G} &= \hat{a}\frac{\bar{a}}{\bar{G}}\bar{G}_a + \hat{b}\frac{\bar{b}}{\bar{G}}\bar{G}_b \\
\hat{G} &= \hat{a}\frac{\bar{a}\bar{q}^a}{\bar{G}} + \hat{b}\frac{\bar{b}\bar{q}^b}{\bar{G}} \\
\hat{G} &= \hat{a}s + \hat{b}(1 - s) && (5) \\
\hat{G}^* &= \hat{a}^*(1 - s) + \hat{b}^* s && (6)
\end{aligned}
$$

The real exchange rate and terms of trade can be written as

$$
\begin{aligned}
e &= \frac{q_a}{q_a^*} = \frac{q_b}{q_b^*} \\
p &= \frac{q_b}{q_a} = \frac{q_b^*}{q_a^*}
\end{aligned}
$$

$$\Downarrow$$

$$\log\frac{e}{\bar{e}} = \log \frac{q_a}{\bar{q}_a}\frac{\bar{q}_a^*}{q_a^*}$$
$$\log\frac{p}{\bar{p}} = \log \frac{q_b}{\bar{q}_b}\frac{\bar{q}_a}{q_a}$$

$$\Downarrow$$

$$\hat{e} = \hat{q}_a - \hat{q}_a^* = \hat{q}_b - \hat{q}_b^* \tag{7}$$
$$\hat{p} = \hat{q}_b - \hat{q}_a = \hat{q}_b^* - \hat{q}_a^* \tag{8}$$

Now combine (4)-(6) and (7) to obtain

$$(1-s)\left(\hat{b}-\hat{a}\right) = \sigma\hat{q}_a \tag{9}$$
$$-s\left(\hat{b}-\hat{a}\right) = \sigma\hat{q}_b = \sigma\left(\hat{e}+\hat{q}_b^*\right)$$

Similarly,

$$s\left(\hat{b}^*-\hat{a}^*\right) = \sigma\hat{q}_a^* = \sigma\left(\hat{q}_a-\hat{e}\right)$$
$$-(1-s)\left(\hat{b}^*-\hat{a}^*\right) = \sigma\hat{q}_b^* \tag{10}$$

Hence

$$-\frac{s}{1-s}\hat{q}_a = \hat{e}+\hat{q}_b^* \tag{11}$$
$$\frac{s}{1-s}\hat{q}_b^* = \hat{e}-\hat{q}_a \tag{12}$$

Combining (9) and (10) gives

$$\hat{q}_a = \frac{s-1}{2s-1}\hat{e} \tag{13}$$
$$\hat{q}_b^* = \frac{1-s}{2s-1}\hat{e} \tag{14}$$

Finally combine (13)-(14) and (8) to obtain

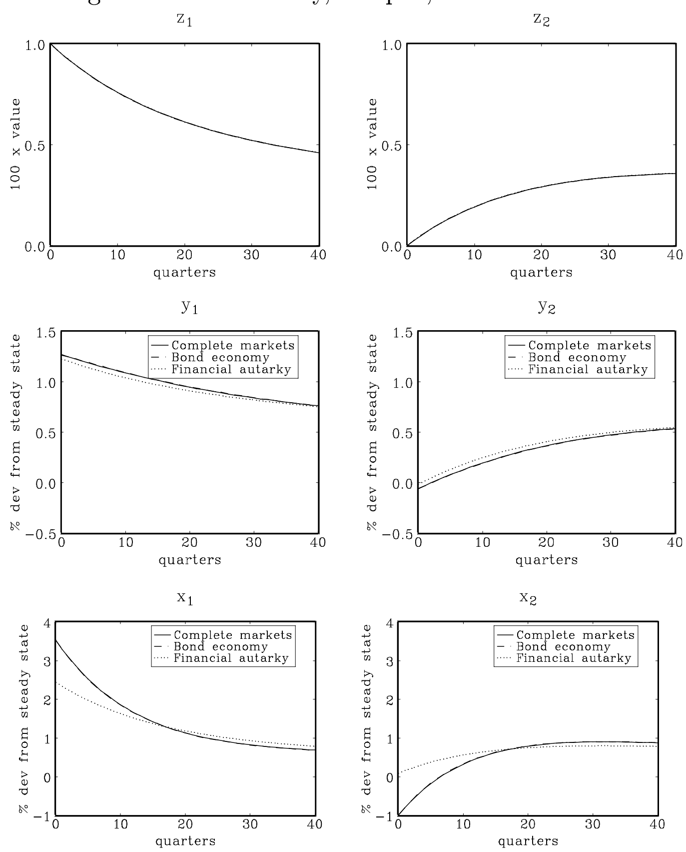
$$\hat{p} = \hat{q}_b - \hat{q}_a = \hat{e}+\hat{q}_b^*-\hat{q}_a = \frac{1}{2s-1}\hat{e} \tag{15}$$

Note that since $s < 1$, the variance of the real exchange rate is always less than the variance of the terms of trade. The opposite is true in the data. (related to the *exchange rate disconnect puzzle*: real exchange rate highly volatile in the data, not in the model)

1.7 Response to productivity shocks

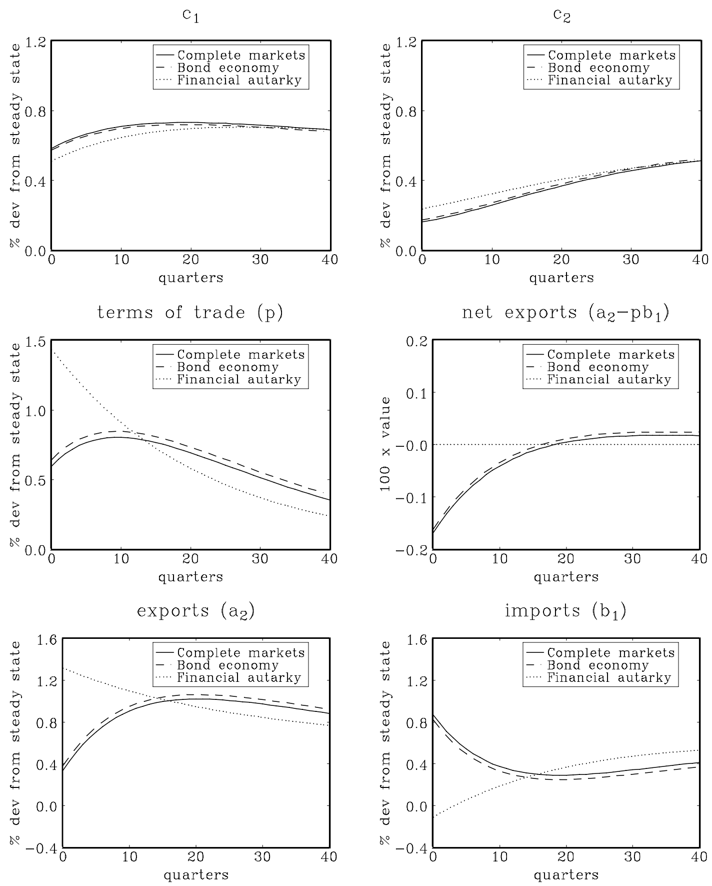
In response to a one percent increase to home productivity, home (foreign) country works more (less) and invests more (less). In financial autarky, investment responds less.

Figure 1: Productivity, Output, and Investment



The productivity increase (and higher home labor supply) leads to a fall in the price of good a. This increases the terms of trade (a deterioration for the home country). Export of good a increases, and in the case of complete markets and bond trading, import of good b also increases since the home country wishes to invest more. The increase in investment is higher than the increase in foreign consumption, leading to a trade deficit. In financial autarky, investment responds less because the home country cannot run a trade deficit to finance more investment.

Figure 2: Consumption and Trade



1.8 Business cycle statistics

This class of models generates countercylical net exports, as in the data. This is in contrast to, for example, a two-country, one good pure exchange economy in which optimal risk sharing implies that an increase in home endowment would lead to a smaller increase in home consumption and thus procyclical net exports. In BKK, risk sharing implies that a positive home productivity shock leads to an increase (decrease) in home (foreign) investment and output, and an increase in consumption in both countries. However domestic investment increases more than foreign consumption, leading to a trade deficit. ("make hay where the sun shines").

B) Correlations with output

Economy	c,y	x,y	n,y	ex,y	im,y	nx,y	p,y	rx,y
				correlation between				
U.S. Data	0.86	0.95	0.87	0.32	0.81	-0.49	-0.24	0.13
Complete markets	0.96	0.96	0.97	0.55	0.89	-0.64	0.65	0.65
Bond economy	0.95	0.96	0.97	0.59	0.86	-0.65	0.65	0.65
Financial autarky	0.92	0.99	0.99	1.00	0.15	0.00	0.65	0.65

P = terms of trade
rx = real exchange rate

Compared to the complete markets and bond economy, the financial autarky model better generates positive international correlations in investment and labor. This is because the channel highlighted above is muted.

C) Cross country correlations and international relative price volatility

	correlation between				*% std dev*	
Economy	y_1,y_2	C_1,c_2	x_1,x_2	N_1,n_2	p	Rx
Data	0.58	0.36	0.30	0.42	2.99	3.73
Complete markets	0.18	0.65	-0.29	-0.14	0.78	0.55
Bond economy	0.17	0.68	-0.29	-0.17	0.84	0.59
Financial autarky	0.24	0.85	0.35	0.14	1.68	1.18

2 Kehoe and Perri (2002)

2.1 Model

One good production model with endogenous incomplete markets.

Planner's objective:

$$\max \left[\lambda_1 \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) U\left(c_1(s^t), \ell_1(s^t)\right) + \lambda_2 \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) U\left(c_2(s^t), \ell_2(s^t)\right) \right]$$

Resource constraints:

$$\sum_{i=1,2} \left[c_i(s^t) + k_i(s^t)\right] = \sum_{i=1,2} \left[F\left(k_i\left(s^{t-1}\right), A_i(s^t)\ell_i(s^t)\right) + (1-\delta)\, k_i(s^{t-1})\right]$$

Enforcement constraints:

$$\sum_{r=t}^{\infty} \sum_{s^r} \beta^{r-t} \pi(s^r|s^t) U\left(c_i(s^r), \ell_i(s^r)\right) \geq V_i\left(k_i(s^{t-1}), s^t\right)$$

Autarky problem:

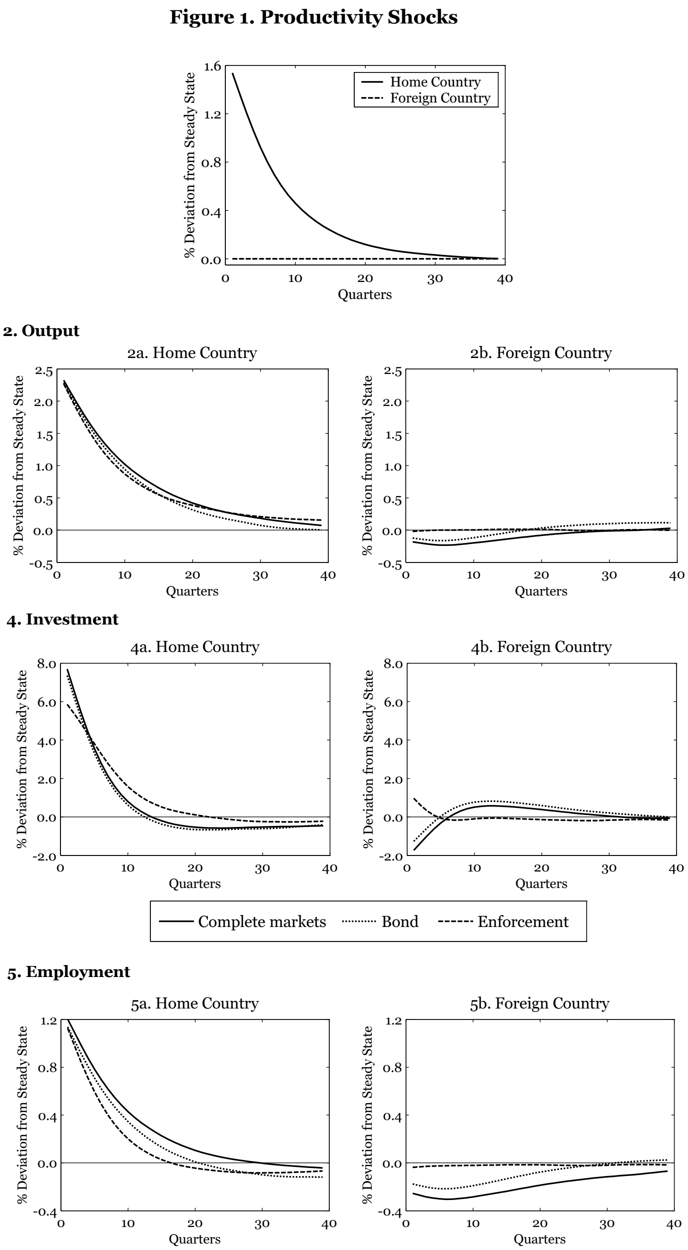
$$V_i\left(k_i(s^{t-1}), s^t\right) = \max \sum_{r=t}^{\infty} \sum_{s^r} \beta^{r-t} \pi(s^r|s^t) U\left(c_i(s^r), \ell_i(s^r)\right)$$
$$\text{s.t. } c_i(s^r) + k_i(s^r) \leq F\left(k_i\left(s^{r-1}\right), A_i(s^r)\ell_i(s^r)\right) + (1-\delta)\, k_i(s^{r-1})$$

Solution method extends Marcet and Marimon (1999).

2.2 Response to productivity shocks

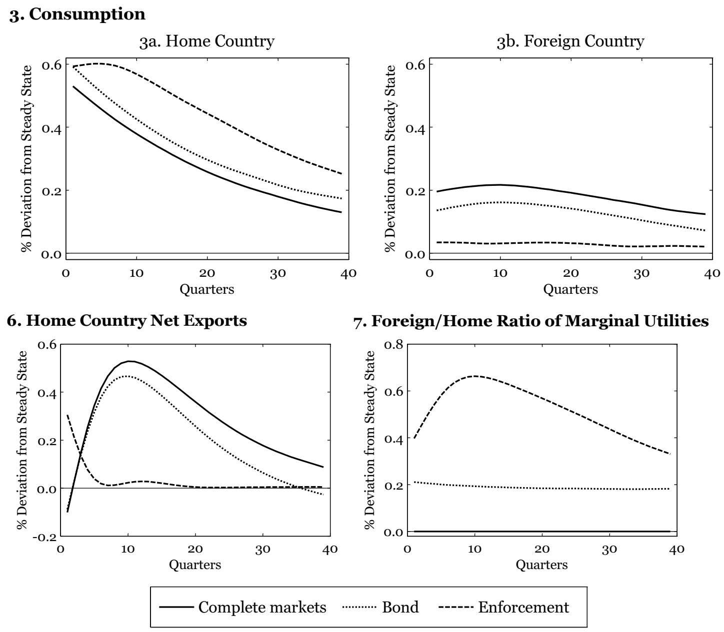
In response to a one percent increase to home productivity, home country works more and invests more. With complete markets, the foreign country works less and invests less, while in the enforcement economy the foreign country also invests more with no change in employment. If the planner shifts investment from foreign to home, this would increase the value of default, the RHS of the enforcement constraint. Instead the planner increases investment in the foreign country to increase the home value of staying in the contract, the LHS of the enforcement constraint.

Figure 3: Productivity, Output, and Investment



In the complete markets economy, risk sharing leads foreign consumption to rise along with the rise in home consumption. Risk sharing is greatly inhibited in the enforcement economy. Because consumption and investment increase in the foreign country without an increase in output, net exports of the home country become positive in the enforcement economy.

Figure 4: Consumption and Trade



2.3 Business cycle statistics

Also included are statistics from Heathcote and Perri (2002) complete markets and financial autarky. The Enforcement model performs well in matching cross country correlations, while getting the cyclicality of net exports wrong. Note that all versions get cross country consumption correlation to be more higher than that of output. (the *consumption correlations puzzle*) The enforcement economy does perform better in that regard.

corr	Data	CM	Enforcement	CM (adj costs)	BKK 94	HP 02
y, y^*	0.51	-0.43	0.25	0.09	0.18	0.24
c, c^*	0.32	0.28	0.29	0.77	0.65	0.85
x, x^*	0.29	-0.99	0.33	-0.09	-0.29	0.35
n, n^*	0.43	-0.58	0.23	-0.15	-0.14	0.14
$\frac{nx}{y}, y$	-0.36	0.06	0.27	-0.02	-0.64	0.00